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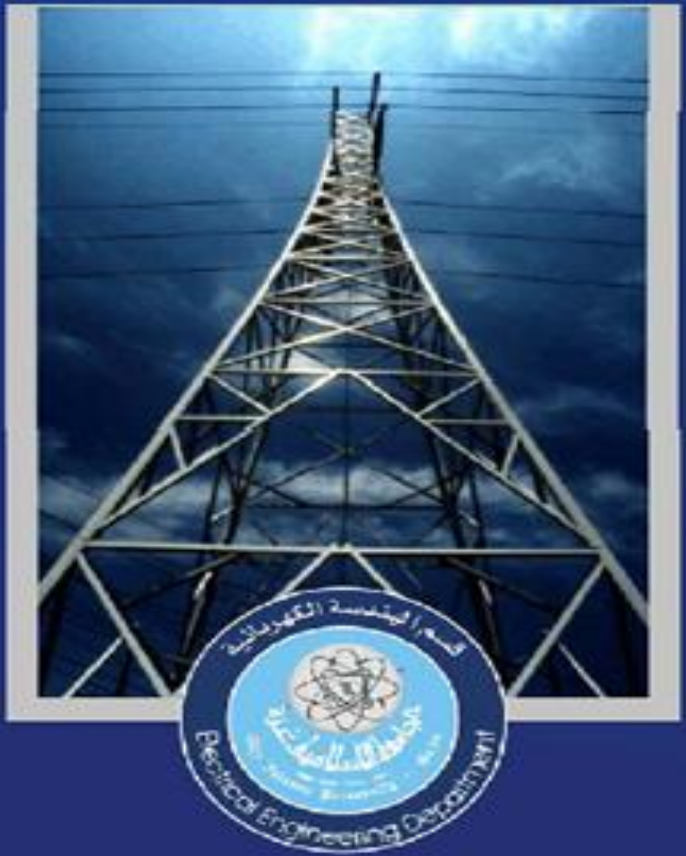

# ELECTRICAL DEPARTMENT

Engineering Faculty

بسم الله الرحمن الرحيم

# PROBABILITY DISCUSSION

## Chapter 1

## Getting Stared with Probability 'PART 2'

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# 1.5 Conditional Probability

**P[A/B] " Probability of A given B "**

$$P[A/B] = \frac{P[AB]}{P[B]}$$

**Notes :**

(1) $P[A/B] \geq P[A]$

(2) If $A \cap B = \phi \rightarrow P[A/B] = 0$

(2) If $B \subset A \rightarrow P[A/B] = \frac{P[AB]}{P[B]} = \frac{P[B]}{P[B]} = 1$

# Quiz 1.5

**Classify the call as:**

***<u>Voice call (V)</u>***

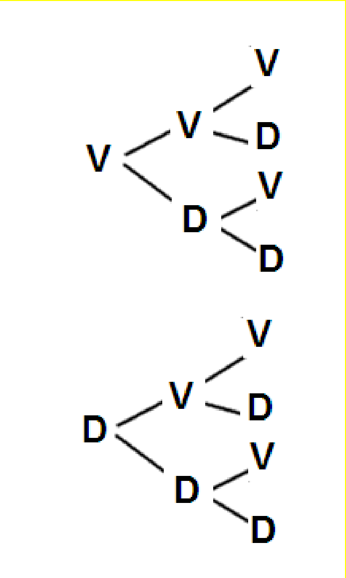
***<u>Data call(D)</u>***

**Observe sequence of three letters.**

**given**

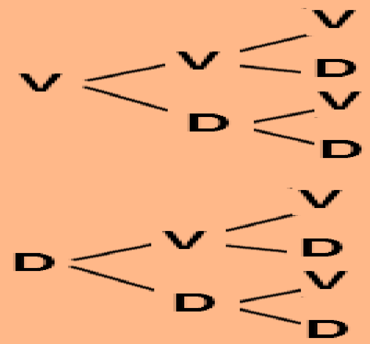
**P[VVV]=P[DDD]=0.2 ,**

**P[VVD]=P[VDV]=P[VDD]=P[DVV]=P[DVD]=P[DDV]=0.1 ,**




**(a) P[Nv=2]=P[{vvd,vdv,dvv}]=0.1+0.1+0.1=0.3**

**(b) P[Nv>=1]=1-P[{ddd}]=1-0.2=0.8**




$$(c)P[\{vvd\}/N_{v=2}]=\frac{P[\{vvd\}.N_{v=2}]}{P[N_{v=2}]}=\frac{P[vvd]}{0.3}=\frac{0.1}{0.3}=\frac{1}{3}$$

$$(d)P[\{ddv\}/N_{v=2}]=\frac{P[\{ddv\}.N_{v=2}]}{P[N_{v=2}]}=\frac{0}{0.3}=0$$

$$(e)P[N_v=2/N_v\geq 1]=\frac{P[N_v=2.N_v\geq 1]}{P[N_v\geq 1]}=\frac{P[N_v=2]}{P[N_v\geq 1]}=\frac{0.3}{0.8}=\frac{3}{8}$$

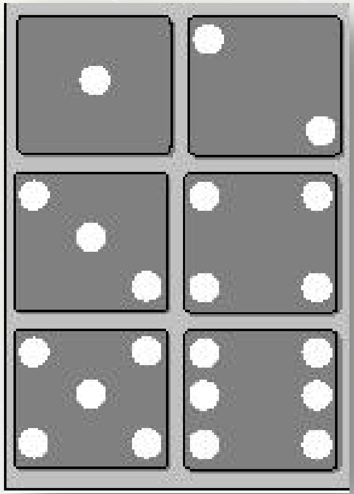
$$(f)P[N_v\geq 1/N_v=2]=\frac{P[N_v\geq 1.N_v=2]}{P[N_v=2]}=\frac{P[N_v=2]}{P[N_v=2]}=1$$

# PROBLEM 1.5.2

Roll six-sided die once . Let
Ri denote the event that the roll is i
Gj denote the event that the roll is grater than j
E denote the event that the roll is even number

(a) What is P[R3/G1] ?

$$P[R3/G1] = \frac{P[R3.G1]}{P[G1]} = \frac{P[R3]}{P[G1]} = \frac{1/6}{5/6} = \frac{1}{5}$$

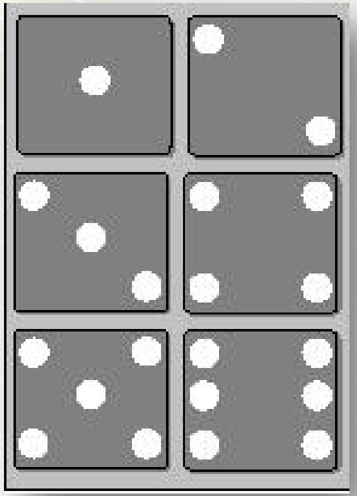


**(b) What is the conditional probability that 6 is rolled given that the roll is grater than 3 ?**

$$R6 = \{6\}$$

$$G3 = \{4,5,6\}$$

$$P[R6/G3] = \frac{P[R6.G3]}{P[G3]} = \frac{P[R6]}{P[G3]} = \frac{1/6}{3/6} = \frac{1}{3}$$

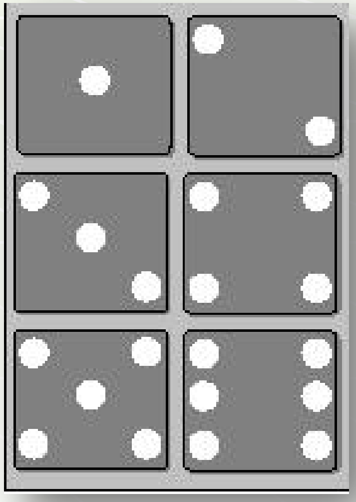


**(c) Given the roll is greater than 3,what is the conditional probability that the roll is even?**

$$G3 = \{4,5,6\}$$

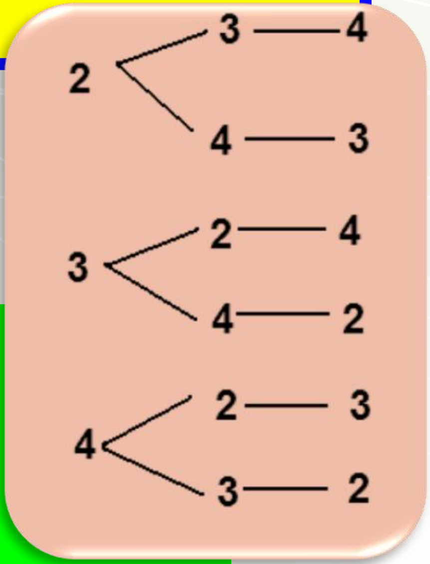
$$E = \{2,4,6\}$$

$$P[G3/E] = \frac{P[G3.E]}{P[E]} = \frac{P[\{4,6\}]}{P[E]} = \frac{2/6}{3/6} = \frac{2}{3}$$



# PROBLEM 1.5.5

**Shuffled deck of three cards:2,3 and 4 .**
**You deal out the three cards.**
**Ei denote the event that the card dealt is even numbered**




**(a) What is P[E2/E1] prob. The 2nd card is even given that the 1st card is even ?**

$$E1=\{234,243,423,432\}$$

$$E2=\{243,342,423,324\}$$

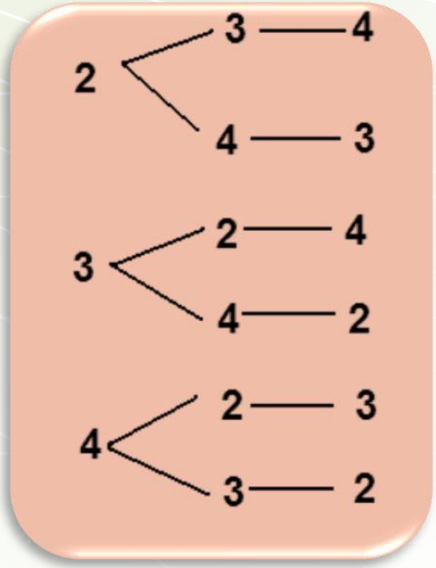
$$P[E2/E1]=\frac{P[E2.E1]}{P[E1]}=\frac{P[\{243,423\}]}{P[E1]}=\frac{2/6}{4/6}=\frac{1}{2}$$

**(b) What is conditional prob. that the first two cards are even given that the third card is even?**

$$E1.E2 = \{243, 423\}$$

$$E3 = \{234, 324, 342, 432\}$$

$$P[E1.E2 / E3] = \frac{P[E1.E2.E3]}{P[E3]} = \frac{P[\phi]}{4/6} = 0$$



# 1.6 Independence

**Events A and B are independent iff:**

$$P[AB]=P[A]\,P[B]$$

$$\boxed{P[A/B]=\frac{P[AB]}{P[B]}=\frac{P[A]P[B]}{P[B]}=P[A]}$$

# Independent Vs disjoint

**Disjoint: $P[AB]=0$**

**Independent: $P[AB]=P[A]\,P[B]$**

# PROBLEM 1.6.1

**Is it possible for events A&B to be independent and satisfy A=B?**

**P[A.B]=P[A.A]=P[A]=P[B] ……….(1)**

**P[A] P[B]=P[A] P[A]=(P[A])^2=(P[B])^2 ……..(2)**

**Satisfy iff :**

**P[A]=1 , A=B=S**

**Or**

**P[A]=0 , A=B=Φ**

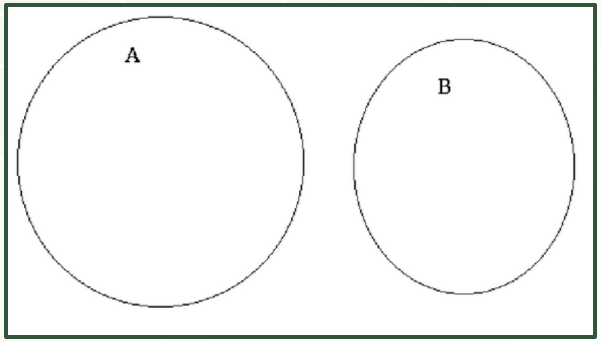
# PROBLEM 1.6.3

**In an experiment A,B,C and D are events with probability:**

**P[A]=1/4 ,,, P[B]=1/8 ,,, P[C]=5/8 ,,, P[D]=3/8**
**A and B are disjoint**
**C and D are independent**
**Find:**

**(a)**

$P[A \wedge B]=0$




$P[A \cup B]=P[A]+P[B]=1/4 + 1/8 =3/8$

$P[A \wedge B^c]=P[A]=1/4$

$P[A \cup B^c]=1-P[B]=1- 1/8 = 7/8$

P[A]=1/4 ,,, P[B]=1/8 ,,, P[C]=5/8 ,,, P[D]=3/8

A and B are disjoint

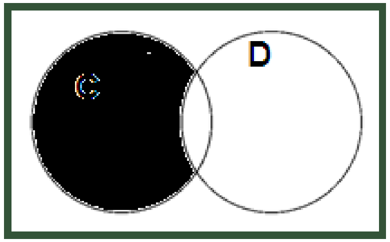
C and D are independent

**(b) Are A and B independent**

P[AB]=0 ➔ Not independent

**(c)**

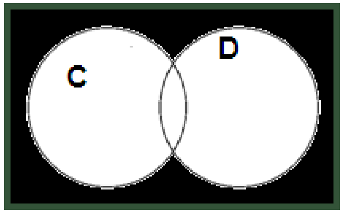
$P[C \wedge D]=P[C]\,P[D]=(5/8)(3/8)=15/64$




$P[C \wedge D^c]=P[C]-P[CD]=(5/8)-(15-64)=25/64$

$P[C^c \wedge D^c]= 1-P[C \cup D]=1-\{P[C]+P[D]-P[CD]\}$

$=1-\{5/8 + 3/8 - 15/64\} = 15/64$



**(d) Are $C^c$ and $D^c$ independent**

$P[C^c \wedge D^c] = 15/64$ ………. (1)

$P[C^c]\,P[D^c] = \{1 - P[C]\}\{1 - P[D]\} = (1 - 5/8)(1 - 3/8)$
$= 15/64$ ……...(2)

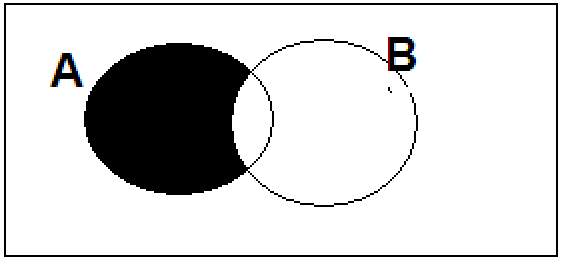
(1)=(2) ➔ Independent

# PROBLEM 1.6.7

**For independent Events A and B prove that:**

**(a) A and $B^c$ are independent.**

**$P[AB]=P[A]P[B]$ …… given**

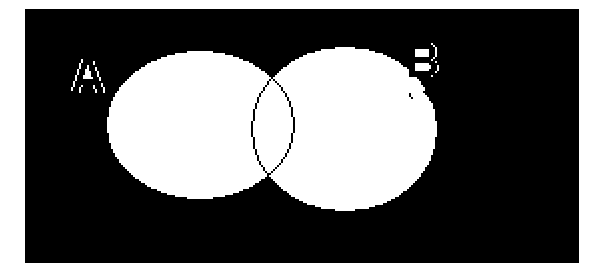



**$P[A B^c]=P[A]-P[AB]=P[A]-P[A]P[B]$**

**$=P[A]\{1-P[B]\} = P[A]\,P[B^c]$**

**➔ A & $B^c$ are indep.**

**(c) $A^c$ and $B^c$ are independent.**




**P[AB]=P[A]P[B] …… given**

$$P[A^c B^c] = P[S] - P[A \cup B]$$

$$= 1 - \{ P[A] + P[B] - P[A]P[B] \}$$

$$= 1 - P[A] - P[B] + P[A]P[B]$$

$$= 1 - P[A] - P[B](1 - P[A])$$

$$= \{ 1 - P[A] \} ( 1 - P[B] )$$

$$= P[A^c]\, P[B^c]$$

**➔ $A^c$ & $B^c$ are indep.**

# PROBLEM 1.6.8

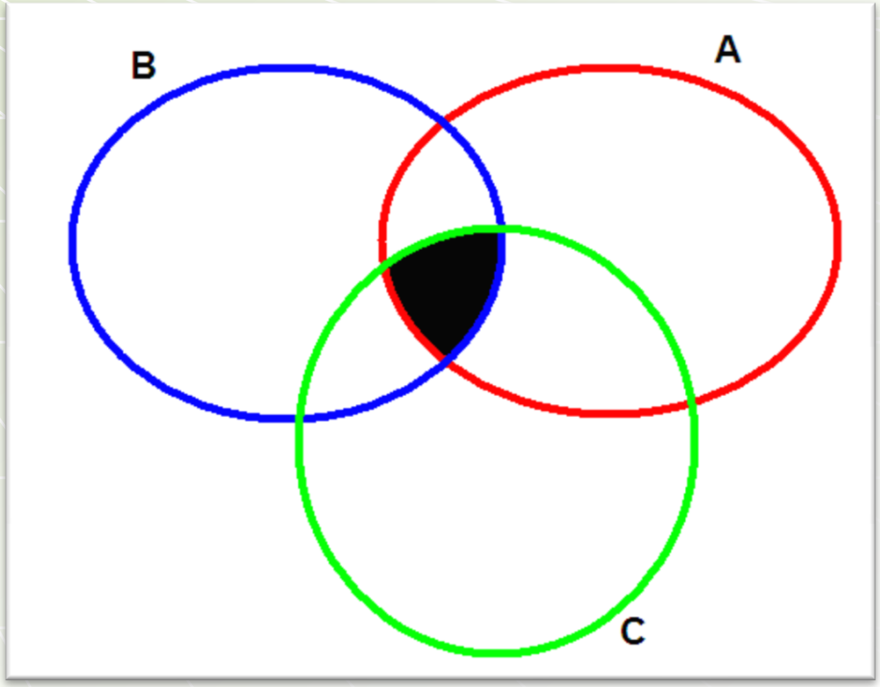
**Use Ven diagram to illustrate 3 events A,B and C that are independent.**

(1) P[AB].P[AC].P[BC]=P[ABC]

(2) P[AB]=P[A]P[B]

(3) P[AC]=P[A]P[C]

(4) P[BC]=P[B]P[C]




Pair wise independent

# PROBLEM 1.6.9

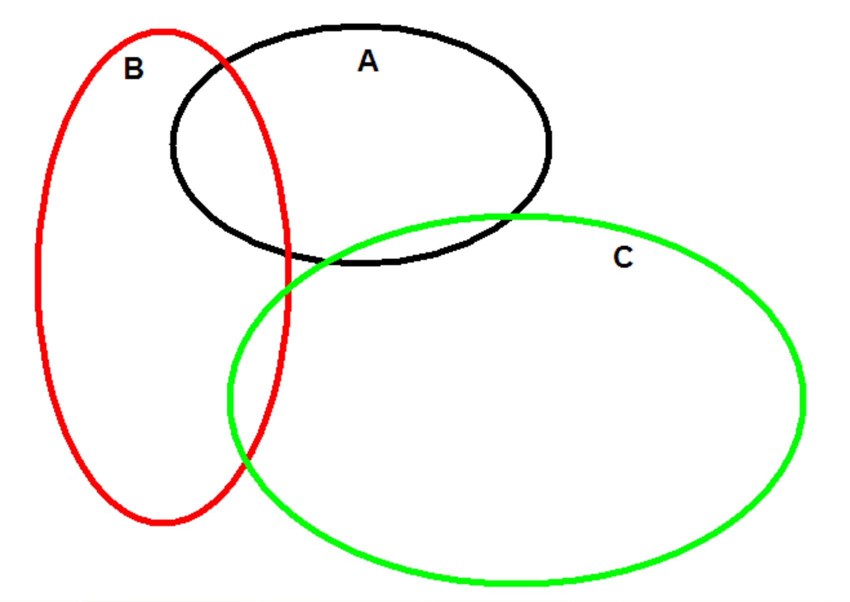
**Use Ven diagram to illustrate 3 events A,B and C that are pair wise independent but not independent .**

**(1) P[AB].P[AC].P[BC] not equal to P[ABC]**

**(2) P[AB]=P[A]P[B]**

**(3) P[AC]=P[A]P[C]**

**(4) P[BC]=P[B]P[C]**



# 1.7 Tree Diagram

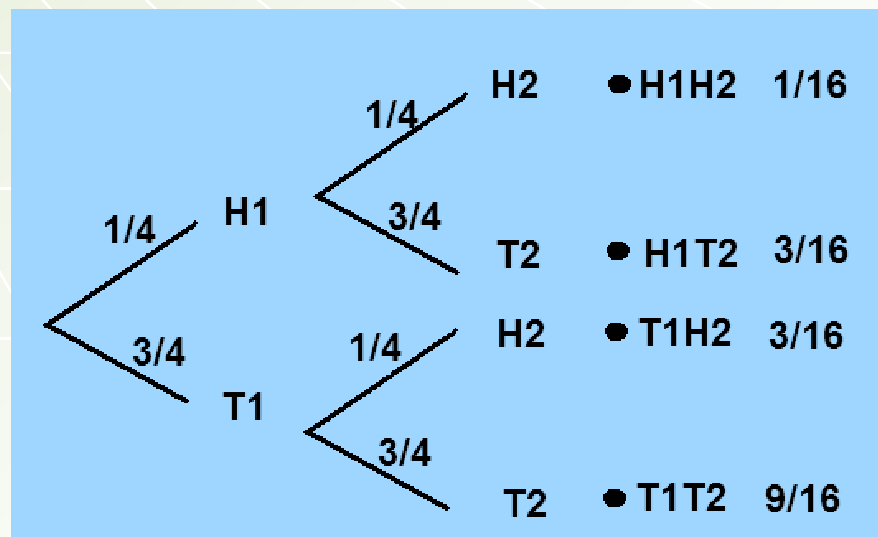
## PROBLEM 1.7.1

**In an experiment flip a coin twice, the coin comes up head with probability ¼ ,**
**Hi and Ti denote the flip of number i**

**(a) What is the probability of P[H1/H2]**

**{{{{ first flip is head given that second flip is head }}}}**

$$P[H1|H2]=\frac{P[H1H2]}{P[H2]}=\frac{\frac{1}{16}}{\frac{1}{16}+\frac{3}{16}}=\frac{1}{4}$$

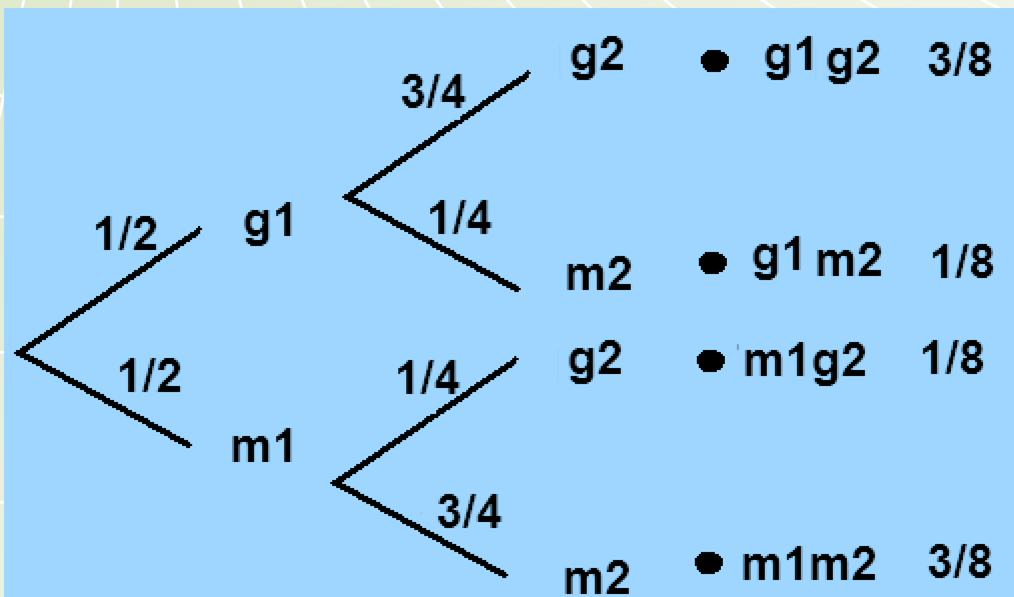



**(b) What is the probability of first flip is head and second is tail**

$$P[H1T2]=3/16$$

# PROBLEM 1.7.3

A basketball team, a player goes to the line for two free throws.
The probability That the first throw is good is ½
If the first is good then the probability that the second is good is ¾ .
However if the first is miss then the second is good with prob ¼
If the player makes exactly one good throw ,the games goes into overtime.
What is the prob. that the game goes into overtime?




$$P[g1m2]+P[m1g2]=1/8+1/8=1/4$$

Equally likely

=Unbiased

=Fair

# PROBLEM 1.7.4

**You have two biased(not fair) coin A&B**

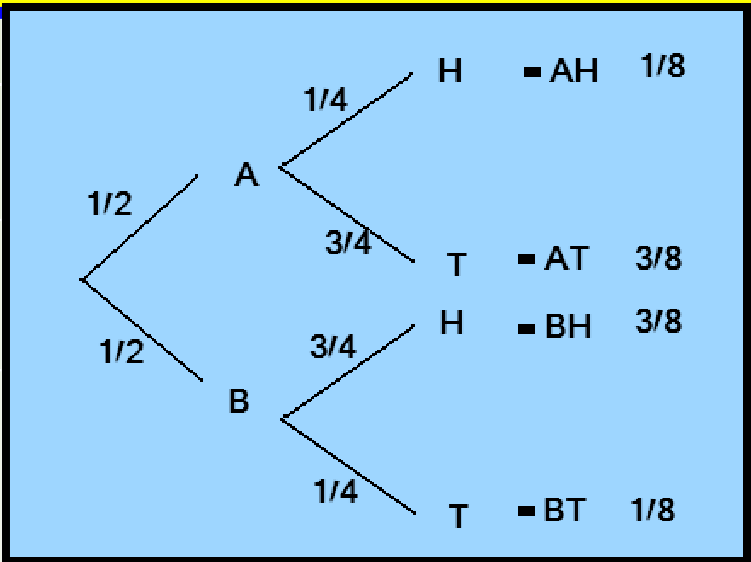
**Coin A comes head with probability ¼ .**

**Coin B comes head with probability ¾ .**

**You chose a coin randomly and you flip it.**

**If the flip is head you guess that the flipped coin is B , otherwise you guess that the flipped coin is A.**

**What is the P[C] ? C:event that your guess is correct**




$$P[C] = P[AT] + P[BH] = 3/8 + 3/8 = 3/4$$

# PROBLEM 1.7.5

For a general population, 1 in 5000 carries the HIV

A test for presence of HIV yields either a positive(+) or negative(-)

Suppose the test gives correct answer 99%

**What is P[-/H] , P[H/+] ?**

**actual**

H → carry HIV

$H^c$ → not carry HIV

**Test results**

+ → carry

- → not carry

$$P[H] = \frac{1}{5000} = 0.0002$$

$$P[H^c] = 1 - 0.002 = 0.9998$$

$$P[-/H] = P[+/H^c] = \%error = 1 - 0.99 = 0.01$$

$$P[-/H] = \frac{P[-.H]}{P[H]} \Rightarrow P[-.H] = 0.002 * 0.01 = 2*10^{-6}$$

$$P[H.+] = 0.0002 - 2*10^{-6}$$

$$= 1.98*10^{-4}$$

| | + | - | |
|---|---|---|---|
| H | 1.98e-4 | 2e-6 | 0.0002 |
| $H^c$ | | | 0.9998 |
| | | | 1 |

$$P[+/H^c]=\frac{P[+.H^c]}{P[H^c]}$$

$$P[+.H^c]=P[+/H^c]*P[H^c]=(0.01)*(0.9998)=9.998*10^{-3}$$

$$P[H/+]=\frac{P[H.+]}{P[+]}=\frac{1.98e-4}{0.010196}=0.019419$$

| | + | - | |
|---|---|---|---|
| H | 1.98e-4 | 2e-6 | 0.002 |
| $H^c$ | 9.998e-3 | 0.989802 | 0.9998 |
| | 0.010196 | 0.989804 | 1 |

# PROBLEM 1.7.7

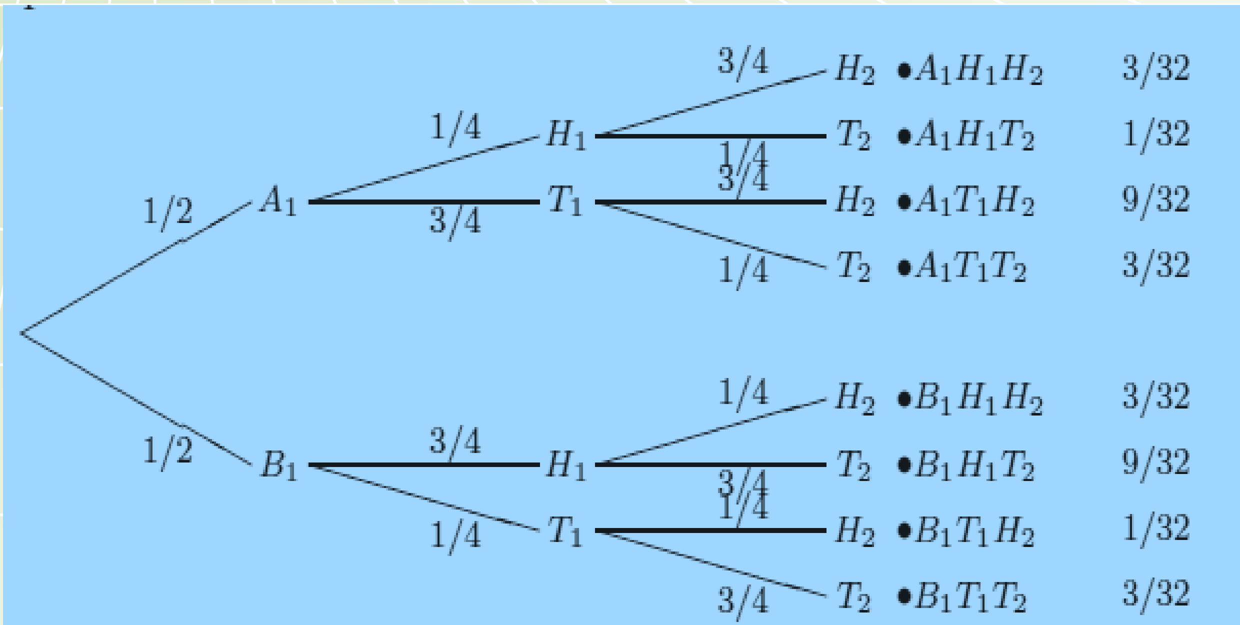
**You have two biased(not fair) coin A&B**

**Coin A comes head with probability ¼ .**

**Coin B comes head with probability ¾ .**

**You chose a coin randomly and you flip it.**

**What is the P[H1H2] ?**




$$P[H1H2] = P[AH1H2] + P[BH1H2] =$$
$$3/32 + 3/32 = 6/32$$

## (b) Are H1 and H2 independent

$$P[H1] = P[AH1H2] + P[AH1T2] + P[BH1H2] + P[BH1T2]$$

$$P[H2] = ......$$

$$P[H1]P[H2] = .....$$

$$P[H1]P[H2] \neq P[H1H2] \Rightarrow dependent$$

# PROBLEM 1.7.8

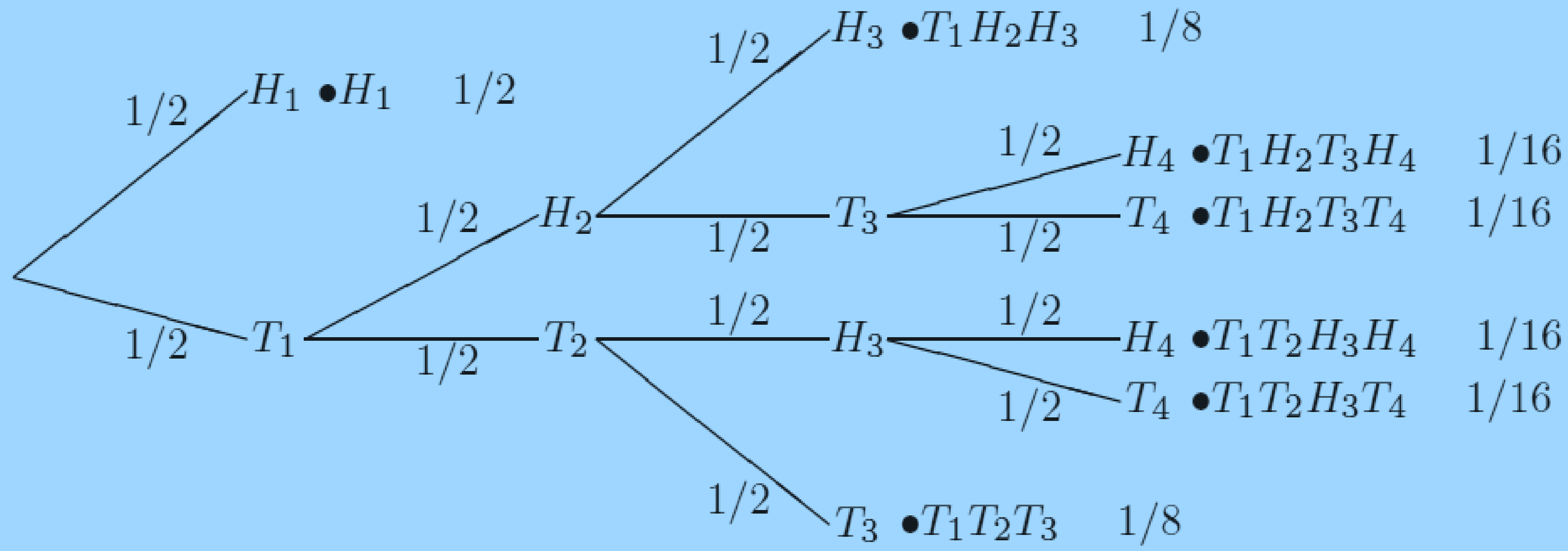
Suppose Dagwood (Blondie's husband) wants to eat a sandwich but needs to go on a diet. So, Dagwood decides to let the flip of a coin determine whether he eats. Using an unbiased coin, Dagwood will postpone the diet (and go directly to the refrigerator) if either (a) he flips heads on his first flip or (b) he flips tails on the first flip but then proceeds to get two heads out of the next three flips. Note that the first flip is *not* counted in the attempt to win two of three and that Dagwood never performs any unnecessary flips. Let $H_i$ be the event that Dagwood flips heads on try $i$. Let $T_i$ be the event that tails occurs on flip $i$.

(a) Sketch the tree for this experiment. Label the probabilities of all outcomes carefully.

(b) What are $P[H_3]$ and $P[T_3]$?

(c) Let $D$ be the event that Dagwood must diet. What is $P[D]$? What is $P[H_1|D]$?

(d) Are $H_3$ and $H_2$ independent events?




$$P[H3] = P[T1H2H3] + P[T1T2H3H4] + P[T1T2H3T4]$$
$$= 1/8 + 1/16 + 1/16 = 1/4$$

$$P[D] = P[T1H2T3T4] + P[T1T2H3T4] + P[T1T2T3] = 1/4$$

$$P[H1/D] = \frac{P[H1.D]}{P[D]} = \frac{P[\varphi]}{P[D]} = 0$$

# PROBLEM 1.7.10

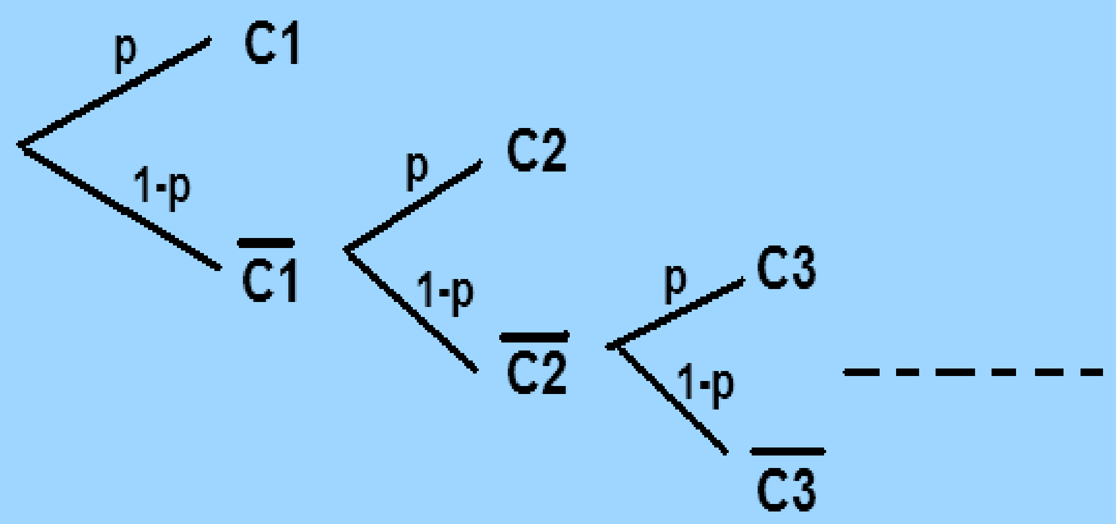
Each time a fisherman casts his line, a fish is caught with probability $p$, independent of whether a fish is caught on any other cast of the line. The fisherman will fish all day until a fish is caught and then he will quit and go home. Let $C_i$ denote the event that on cast $i$ the fisherman catches a fish. Draw the tree for this experiment and find $P[C_1]$, $P[C_2]$, and $P[C_n]$.

$$P[C1] = P$$

$$P[C2] = (1 - P)P$$

$$P[C3] = (1 - p)^2 P$$

$$P[Cn] = (1 - p)^{n-1} P$$



try to solve : 1.7.6,1.7.9

# Thank You !

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